Nonlinear medium-term hydro-thermal scheduling with transmission constraints

Leonardo Martins
leonardo.martins@br.ibm.com

IBM Research

1st Brazilian Workshop on Interior-Point Methods

April 28th, 2015

Summary

- In collaboration[1] with Anibal Azevedo and Secundino Soares, University of Campinas (UNICAMP)
- Practical experience with primal-dual interior-point algorithm with logarithmic barrier function with Wächter and Biegler's line search filter[2] method (IPOPT)
- Filters have shown improved numerical convergence over penalty functions

[1] MARTINS, L. S. A.; AZEVEDO, A. T.; SOARES, S., "Nonlinear medium-term hydro-thermal scheduling with transmission constraints," IEEE Trans. Power Systems, 29(4), pp. 1623-1633, 2014

[2] WÄCHTER, A.; BIEGLER, L. T., "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," Math. Program. Ser. A, 106(1), pp. 25-57, 2006

Hydro-thermal scheduling

Definition

- Determine the use of water resources for electricity generation in power systems over a period of time such that thermal power generation costs are minimized
- At every time interval j and power balance group k, the sum of hydro power generation and that from other sources must meet load demand

$$d_{k,j} = \sum_{\forall i \in \mathcal{I}_k} p_{i,j}(\cdot) + \sum_{\forall t \in \mathcal{T}_k} z_{t,j} \tag{1}$$

- Hydro power is produced with negligible variable costs

Hydroelectric power

World

- 20% of world's electricity supply
- 30 countries
- 90% of economically feasible potential in developing countries

Brazil

- About 80 GW of installed capacity
- 150+ plants with > 30 MW
- 69% share
- 56% capacity factor
- Over 1,000 plants
- 13 river basins

Hydro-thermal scheduling in Brazil

Fact sheet

- Installed capacity of all sources is over 120 GW
- Hydro accounts for about 87% of power generation
- More than 96% of the system is interconnected
- Large reservoirs with seasonal regulation capacities
- Considerable head variation as a ratio of total head

Market clearing

- Central coordination and control
- Sets weekly wholesale energy prices in the Brazilian power system for three load levels
- Deterministic linear programming with individual plant representation
- Coupling with SDDP by means of cost-to-go functions

Hydroelectric production function

- Nonlinear function of turbine and generator efficiencies, water discharge and head

$$p_j(\cdot) = \varepsilon\left(q_j, h_j\right) \cdot q_j \cdot h_j \tag{2}$$

where water head is a function of forebay and tailrace elevations, and penstock loss

$$h_j(\cdot) = \phi\left(a_j\right) - \theta\left(r_j\right) - \tau\left(q_j\right) \tag{3}$$

- Water release $r_j = \sum_{l \in \mathcal{L}_j} q_{l,j} + v_j$
- Forebay, tailrace elevations and penstock loss are polynomials

Linear v. nonlinear

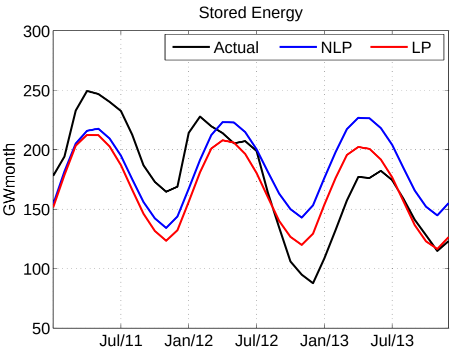

Figure: Simulation results[3]

[3] ZAMBELLI, M. S.; MARTINS, L. S. A.; SOARES, S., "Model predictive control applied to the long-term hydrothermal scheduling of the Brazilian power system," IEEE Powertech Grenoble, 2013

Transmission constraints[4]

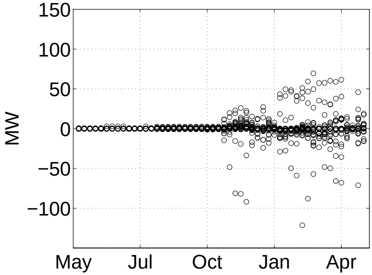

Figure: Transmission use

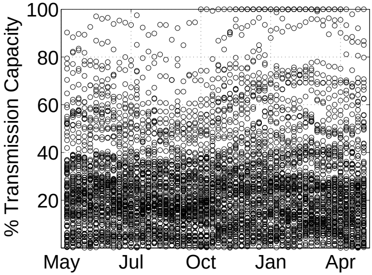

Figure: Output differences

[4] FUJISAWA, C. H.; MARTINS, L. S. A.; AZEVEDO, A. T.; SOARES, S., "Equivalent networks for medium-term hydro-thermal scheduling with variable head cascaded reservoirs and transmission constraints," Power Systems Computation Conference (PSCC), 2014

Mathematical formulation

Let the hydroelectric production scheduling in systems with central coordination be formulated as follows

$$\min_{y,f,z} \quad \Psi(z) \tag{4}$$

$$\text{subject to } Ay = b \tag{5}$$

$$Bf = Hp(y) + Gz - d \tag{6}$$

$$CXf = \mathbf{0} \tag{7}$$

$$y \in \mathcal{Y} \tag{8}$$

$$f \in \mathcal{F} \tag{9}$$

$$z \in \mathcal{Z} \tag{10}$$

Variables

- Indexed over time, load level, and space (in this order)
- Hydro operation y is decomposed into volume of stored water a, and water discharge q and spillage v, such that

$$y = \begin{pmatrix} a \\ q \\ v \end{pmatrix} \in \mathbb{R}^{n_y}$$

- Power flow is given by $f \in \mathbb{R}^{n_f}$
- Thermal power generation is given by $z \in \mathbb{R}^{n_z}$

Objective function

Definition

Minimize thermal power generation costs over planning horizon

$$\Psi(z) = \sum_{\forall j \in \mathcal{J}} \sum_{\forall l \in \mathcal{L}_j} \sum_{\forall t \in \mathcal{T}_j} \psi_{t,l,j}\left(z_{t,l,j}\right)$$

where $z_{t,l,j}$ is thermal power generation at plant t for load level l at stage j

Notes

- Separable, twice-differentiable, and convex

Constraints

- Water conservation is described by $g^{\alpha} : \mathbb{R}^{n_y} \to \mathbb{R}^{m_\alpha}$ over y:

$$g^{\alpha}(y) = Ay - b \tag{11}$$

- Power balance is given by $g^{\beta} : \mathbb{R}^{n} \to \mathbb{R}^{m_\beta}$ over y, f, z

$$g^{\beta}(y, f, z) = Hp(y) + Gz - Bf - d \tag{12}$$

- Kirchhoff's Second Law is defined by $g^{\gamma} : \mathbb{R}^{n_f} \to \mathbb{R}^{m_\gamma}$ over f representing the null sum of phase angle differences for every basic loop

$$g^{\gamma}(f) = CXf \tag{13}$$

Notes

- CX is a block-diagonal network loop reactance matrix, where every row of C describes a loop in B, and X has reactance values in the diagonal

First-Order Optimality Conditions

- The Lagrangian is given as follows:

$$\ell_\mu(\tilde{w}, \lambda, \pi, \zeta) = \varphi_\mu(\tilde{w}) + g(w)^T \lambda + (w + s - u)^T \pi + (t - w + l)^T \zeta$$

- For which we have the first-order optimality conditions:

$$L_\mu(\tilde{w}, \lambda, \pi, \zeta) \triangleq \begin{pmatrix} \nabla f(w) + \nabla g(w)^T \lambda + \pi - \zeta \\ g(w) \\ w + s - u \\ t - w + l \\ \pi - \mu S^{-1} e \\ \zeta - \mu T^{-1} e \end{pmatrix} = \mathbf{0}$$

Primal Search Direction

$$
\begin{aligned}
D_y \Delta y &= \varrho^y - A^T \Delta\lambda^\alpha + \left(H \nabla p(y)\right)^T \Delta\lambda^\beta \\
D_f \Delta f &= \varrho^f - B^T \Delta\lambda^\beta - (CX)^T \Delta\lambda^\gamma \\
D_z \Delta z &= \varrho^z + G^T \Delta\lambda^\beta
\end{aligned}
$$

- D_y is tridiagonal
- D_f, D_z are diagonal

Dual Search Direction

$$\Lambda\Delta\lambda^\alpha = AD_y^{-1}\left(\varrho^y + \left(H\nabla p(y)\right)^T \Delta\lambda^\beta\right) - \rho_\lambda^\alpha$$

$$\Gamma\Delta\lambda^\gamma = CXD_f^{-1}\left(\varrho^f - B^T\Delta\lambda^\beta\right) - \rho_\lambda^\gamma$$

$$\begin{aligned}\Upsilon\Delta\lambda^\beta \quad = \quad & \rho_\lambda^\beta + H\nabla p(y)D_y^{-1}A^T\Lambda^{-1}\ldots \\ & \left(\rho_\lambda^\alpha - AD_y^{-1}\varrho^y\right) - BD_f^{-1}(CX)^T\Gamma^{-1}\ldots \\ & \left(\rho_\lambda^\gamma - CXD_f^{-1}\varrho^f\right) + H\nabla p(y)D_y^{-1}\varrho^y - \ldots \\ & -BD_f^{-1}\varrho^f + GD_z^{-1}\varrho^z\end{aligned}$$

- Both $\Lambda = AD_y^{-1}A^T$ and $\Gamma = CXD_f^{-1}(CX)^T$ have block-diagonal structures

Line Search

- A step length $\alpha_p^{(k)} \in \left[\underline{\alpha}_p^{(k)}, \overline{\alpha}_p^{(k)}\right]$ is calculated
- It is accepted if **at least one** of the following conditions is satisfied

$$
\begin{aligned}
\theta(\tilde{w}^{(k+1)}) &\leq (1-\gamma_\theta)\theta(\tilde{w}^{(k)}) \\
\varphi_{\mu^{(k)}}(\tilde{w}^{(k+1)}) &\leq \varphi_{\mu^{(k)}}(\tilde{w}^{(k)}) - \gamma_\varphi \theta(\tilde{w}^{(k)})
\end{aligned}
$$

  where $\theta(\tilde{w}) = \|\tilde{g}(\tilde{w})\|_2$
- Close to convergence, Armijo's rule must be satisfied
- In all cases, $(\theta(\tilde{w}^{(k+1)}), \varphi(\tilde{w}^{(k+1)})) \notin F^{(k)}$

Filter

- A set $F^{(k)} \subseteq \{(\theta, \varphi) \in \mathbb{R}^2 : \theta \geq 0\}$
- Created empty such that

$$F^{(0)} \leftarrow \{(\theta, \varphi) \in \mathbb{R}^2 : \theta \geq \theta_{\max}\}$$

- At every iteration it is augmented such that

$$\begin{aligned} F^{(k+1)} \quad \leftarrow \quad & F^{(k)} \cup \{(\theta, \varphi) \in \mathbb{R}^2 : \\ & \quad \theta \geq (1 - \gamma_\theta)\theta(\tilde{w}^{(k)}) \text{ and} \\ & \quad \varphi \geq \varphi_{\mu^{(k)}}(\tilde{w}^{(k)}) - \gamma_\varphi \theta(\tilde{w}^{(k)})\} \end{aligned}$$

Line search filter in practice

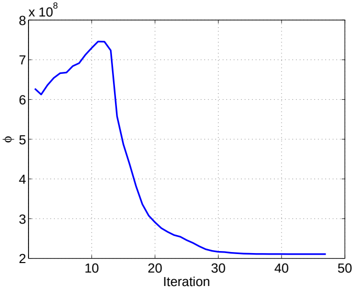

Figure: Objective function

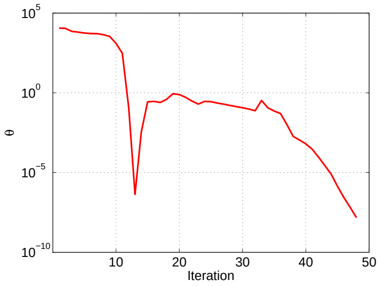

Figure: Constraint violation

Conclusion

- Successful global convergence to local minima on all problem instances for official data publicly available
- Good computational performance, e.g. 2 minutes for 60k variables and 30k equations

Next steps

- Conic relaxation of nonconvex quadratic approximations

Thank You!